

MATH 3012G Test III

Spring 2010

Name: _____

GTid (9xxxxxxx): _____

Group: _____

Instructor: Mitchel T. Keller

There are 6 questions on this exam on 4 pages (not counting this coverage). **Answer each question in the space provided. If you need additional space, additional pages will be provided. The back of pages will not be graded!** Be sure to explain your answers, as answers that are not accompanied by explanations/work may receive no credit. **Use complete sentences wherever possible;** answers that do not contain at least one complete sentence of explanation (and do not just ask for a list or for you to label something or run an algorithm) will not receive full credit. Place your name and group on each page. Any page missing any of this information will **not** be graded.

You are to complete this exam completely alone, without the aid of notes, texts, calculators, cellular telephones, personal digital assistants, or any other mechanical or digital calculating device.

By signing on the line below, you agree to abide by the Georgia Tech Honor Code and Student Code of Conduct, the principles of which are embodied by the Challenge Statement:

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Student signature: _____

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
Total:	30	

1. (5 points) A businesswoman is traveling in Belgium and wants to buy chocolates for herself, her husband, and their two daughters. A store has dark chocolate truffles (€10/box), milk chocolate truffles (€8/box), nougat-filled chocolates (€5/box), milk chocolate bars (€7/bar), and 75% cacao chocolate bars (€11/bar). Her purchase is to be subject to the following:
- Only the daughters like dark chocolate truffles, and her purchase must ensure that each daughter gets an equal number of boxes of them (if they get any).
 - At least two boxes of milk chocolate truffles must be purchased.
 - If she buys any boxes of nougat-filled chocolates, then she buys exactly enough that each family member gets precisely one box of them.
 - At most three milk chocolate bars may be purchased.
 - There are no restrictions on the number of 75% cacao chocolate bars.

Give a generating function (either as a quotient of polynomials or product of polynomials and infinite series) in which the coefficient on x^n is the number of ways the woman can spend exactly € n at this chocolate shop. You do **not** need to find a formula for this coefficient.

2. (5 points) Give the general solution to the homogeneous advancement operator equation

$$(A - 3)^3(A - 1)^4(A + 2)(A - 5)f = 0.$$

3. (5 points) A valid database identifier of length n can be constructed in three ways:

- Starting with A and followed by any valid identifier of length $n - 1$.
- Starting with one of the two-character strings $1A, 1B, 1C, 1D, 1E,$ or $1F$ and followed by any valid identifier of length $n - 2$.
- Starting with 0 and followed by any ternary ($\{0, 1, 2\}$) string of length $n - 1$.

Find a recurrence for the number $g(n)$ of database identifiers of length n and then solve your recurrence to obtain an explicit formula for $g(n)$. (You may use the initial condition $g(0) = 1$ in your work.)

4. (5 points) The table to the right contains the weight of the directed edge (x, y) in the intersection of **row** x and **column** y in a digraph with vertex set $\{A, B, C, D, E, F\}$. For example, $W(B, D) = 21$. (On the other hand, $W(D, B) = 10$.) Use this data and Dijkstra's algorithm to find the distance from A to each of the other vertices and a path of that length from A .

W	A	B	C	D	E	F
A	0	12	8	43	79	35
B	93	0	18	21	60	33
C	17	3	0	37	50	30
D	85	10	91	0	17	7
E	28	47	39	14	0	108
F	31	7	29	73	20	0

5. (5 points) Consider a network in which the source S has precisely three neighbors: $B, E,$ and F . Suppose also that $c(S, B) = 30, c(S, E) = 20,$ and $c(S, F) = 25$. You know that there is a flow ϕ on the network but you do not know how much flow is on any edge. You do know, however, that when the Ford-Fulkerson labeling algorithm is run on the network with current flow ϕ , the first two vertices labeled are S with label $(*, +, \infty)$ and F with label $(S, +, 15)$. Use this information to determine the value of the flow ϕ and explain how you do so.

6. (5 points) Use the Ford-Fulkerson labeling algorithm to find a maximum flow ϕ , the value of ϕ , and a cut of capacity equal to that value in the network on the solution sheet. Update the diagram to show the maximum flow and explain how the cut's capacity is determined.

