MATH 1501G1/G2/G3 Final Exam

Fall 2006

Name: _____

GTid (9xxxxxxxx): _____

Instructor: Mitchel T. Keller

Teaching Assistant and Section: _____

There are 14 questions on this exam on 2 pages (not counting this coverpage). Answer each question on a separate solution sheet (you may use more than one solution sheet per problem if needed). Be sure to explain your answers, as answers that are not accompanied by explanations/work may receive no credit. Place your name, section, and problem number on each solution sheet. Any solution sheet missing any of this information will **not** be graded.

You are to complete this exam completely alone, without the aid of notes, texts, calculators, cellular telephones, personal digital assistants, or any other mechanical or digital calculating device.

By signing on the line below, you agree to abide by the Georgia Tech Honor Code, the principles of which are embodied by the Challenge Statement:

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Failure to sign this cover page will *not* be considered evidence of academic misconduct. However, **if the cover page is not signed, seven points will be deducted from your raw total score on this exam.**

Student signature: _____

1. (5 points) Find the limit
$$\lim_{x\to 0}\left[6x^2(\cot x)(\csc(2x))\right].$$

2. (5 points) Consider the sequence
$$a_n = \cos\left(\frac{2\pi}{n}\right).$$
Is a_n bounded? If so, give upper and lower bounds (as appropriate). Is a_n monotonic? Explain why or why not. If a_n converges, find its limit; if it doesn't, give a brief explanation why it does not converge.

3. (5 points) Use the definition of the derivative to show that the function $f(x) = 1/x^2$ has derivative $-2/x^3$ for $x \neq 0$.

4. (5 points) Find $f'(x)$ if
$$f(x) = x\tan(x) + \sin(3x^{4/5}) + \frac{x^2+1}{\ln(x^2+1)}.$$

5. (5 points) You haul a 10 pound bucket 30 feet up a well. You begin with 100 pounds of water in the bucket, but it is leaking at a constant rate so that you lose 2 pounds of water for every foot that you pull it up. If the rope weighs $\frac{1}{2}$ lb per foot, how much work is done in bringing the bucket of water to the top of the well?

6. (5 points) Find $f'(x)$ if
$$f(x) = \int_{4x^2}^{2\pi+e} \sqrt{\ln|\arctan(t^2)|}\,dt.$$

7. (5 points) Let $z = 4+2i$ and $w = 1-i$. Write the complex number below in rectangular form.
$$2e^{i(\pi/3)} + \frac{(\overline{z})}{w} + |z-w|$$

8. (5 points) Let Ω be the region bounded by the curves $y = e^{2x}$, $x = 1$, $x = 2$, and $y = 0$. Find the volume of the solid obtained by revolving Ω about the y-axis.

9. (5 points) A can is to be constructed in the shape of a right circular cylinder, and it will hold 500π cubic inches. Find the minimal cost for constructing such a cylindrical can if the cost of constructing the top and bottom is 4 cents per square inch and the cost of the cylindrical "side" is 2 cents per square inch.

10. (5 points) Evaluate the definite integral $\displaystyle\int_1^2 \frac{1}{x^2\sqrt{4x^2+1}}\,dx$.

11. (5 points) Two coal carts which ride on the same set of railroad tracks are attached by 33 feet of rope. The rope is pulled taut and threaded through a pulley at the top of a 12 foot tall pole; the pole is located in the middle of the tracks between the two carts. If one cart is moving away from the pole at a speed of 3 feet per second when it is 5 feet from the pole, at what speed is the other cart approaching the pole?

12. (5 points) Evaluate $\displaystyle\int \frac{x^2+3x}{(x-1)(x^2+1)}\,dx$.

13. (5 points) Give a δ, ε proof that

$$\lim_{x\to 3}(-2x+5) = -1.$$

14. (5 points) Below is a list of properties of a function f. On the special solution sheet with coordinate axes drawn on it, sketch the graph of a function f with those properties. [*Hint*: Use the information given to identify x-intercepts, horizontal asymptotes, vertical asymptotes, where f is increasing, where f is decreasing, local maxima, local minima, and concavity. Then sketch the graph.]

    1. $f(-3) = f(-1) = f(1) = f(3) = 0$
    2. f is differentiable for all x except $x = 2$
    3. $f'(-2) = f'(0) = 0$
    4. $f'(x) > 0$ on $(-2, 0)$
    5. $f'(x) < 0$ on $(-\infty, -2)$, $(0, 2)$, and $(2, +\infty)$
    6. $f''(-3) = f''(-1) = 0$
    7. f is concave up on $(-3, -1)$ and $(2, +\infty)$
    8. f is concave down on $(-\infty, -3)$ and $(-1, 2)$
    9. $f(x) \to +\infty$ as $x \to 2^+$
    10. $f(x) \to -\infty$ as $x \to 2^-$
    11. $\lim\limits_{x\to\infty} f(x) = -2$
    12. $\lim\limits_{x\to-\infty} f(x) = 2$