MATH 1501G1/G2/G3 **Name:** ____________________

Quiz #3 7 September 2006

1. (5 points) For the function $f(x)$ below, classify each dicontinuity (if there are any) as being a removable discontinuity, a jump discontinuity, or an infinite discontinuity and explain *why* it is that type of discontinuity. Where possible, define $f(x)$ at the points of discontinuity so that $f(x)$ becomes continuous at those points. If it is not possible to do so, write "not possible."

$$f(x) = \begin{cases} \frac{x-1}{x+1} & x < -1; \\ \frac{\sin(3x)}{2x} & -1 \le x < 0; \\ 3x+6 & 0 \le x < 2; \\ x^2+4x & 2 \le x < 3; \\ 14 & x = 3; \\ 7x & 3 < x \end{cases}$$

Solution: We see that f is discontinuous at $x = -1$, since the left-hand limit there does not exist (f tends to $-\infty$ from the left there) but the limit from the right is $\sin(-3)/-2$. Thus, there is an infinite discontinuity at $x = -1$, and we cannot make f continuous there.

At $x = 0$, we see that the left-hand limit is

$$\lim_{x\to 0^-} \frac{\sin(3x)}{2x} = \lim_{x\to 0^-} \left[\frac{3}{2}\frac{\sin(3x)}{3x}\right] = \frac{3}{2}.$$

However, the right-hand limit is 6, so f has a jump discontinuity at $x = 0$, and therefore it is not possible to make it continuous there.

Notice that $\lim_{x\to 2^-} f(x) = 12 = \lim_{x\to 2^+} f(x)$, so f is continuous at $x = 2$.

At $x = 3$, we have $\lim_{x\to 3^-} f(x) = 21$ and $\lim_{x\to 3^+} f(x) = 21$. However, $f(3) = 14$. Thus, f is not continuous at $x = 3$ and it has a removable discontinuity there. We can make f continuous by defining $f(3) = 21$.