1. (5 points) Compute the derivative

$$\frac{d}{dx}\left(\frac{x^3+x^2+x-1}{x^3-x^2+x+1}\right)$$

and evaluate it at $x = 1$. (You do not need to simplify your expression for the derivative.)

Solution: We compute the derivative using the quotient rule and have

$$\begin{aligned}
&\frac{d}{dx}\left(\frac{x^3+x^2+x-1}{x^3-x^2+x+1}\right)\\
&\quad= \frac{(x^3-x^2+x+1)\frac{d}{dx}(x^3+x^2+x-1)-(x^3+x^2+x-1)\frac{d}{dx}(x^3-x^2+x+1)}{(x^3-x^2+x+1)^2}\\
&\quad= \frac{(x^3-x^2+x+1)(3x^2+2x+1)-(x^3+x^2+x-1)(3x^2-2x+1)}{(x^3-x^2+x+1)^2}\\
&\quad= \frac{3x^5+2x^4+x^3-3x^4-2x^3-x^2+3x^3+2x^2+x+3x^2+2x+1}{(x^3-x^2+x+1)^2}\\
&\qquad- \frac{3x^5-2x^4+x^3+3x^4-2x^3+x^2+3x^3-2x^2+x-3x^2+2x-1}{(x^3-x^2+x+1)^2}\\
&\quad= \frac{4x^4-6x^4-2x^2+4x^2+6x^2+2}{(x^3-x^2+x+1)^2}\\
&\quad= \frac{-2x^4+8x^2+2}{(x^3-x^2+x+1)^2}
\end{aligned}$$

(It'd be just fine for you to stop simplifying at the end of the third line above.) When $x = 1$, this derivative is thus

$$\frac{-2+8+2}{(1-1+1+1)^2} = \frac{8}{4} = 2.$$