1. (5 points) Take the Derivative using the Chain Rule

$$y = \tan^3\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right) + \frac{1}{\sqrt{1-\sin^2(x^2)}}.$$

Solution: We begin by noting that $1 - \sin^2(x^2) = \cos(x^2)$, so the second term becomes $1/\cos(x^2) = \big(\cos(x^2)\big)^{-1}$. We now start differentiating:

$$\begin{aligned}
\frac{dy}{dx} =& 3\tan^2\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right)\frac{d}{dx}\left(\tan\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right)\right) \\
&+ (-1)(\cos(x^2))^{-2}\frac{d}{dx}(\cos(x^2)) \\
=& 3\tan^2\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right)\sec^2\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right)\frac{d}{dx}\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right) \\
&+ (-1)(\cos(x^2))^{-2}(-\sin(x^2)(2x)) \\
=& 3\tan^2\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right)\sec^2\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right)\left(\frac{1}{2\sqrt{\sin(2x)}}\frac{d}{dx}\sin(2x)\right. \\
&\left.+ \frac{3\pi}{2}(x+4)^{1/2}\right) + (-1)(\cos(x^2))^{-2}(-\sin(x^2)(2x)) \\
=& 3\tan^2\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right)\sec^2\left(\sqrt{\sin(2x)} + \pi(x+4)^{3/2}\right)\left(\frac{1}{2\sqrt{\sin(2x)}}\cos(2x)\cdot 2\right. \\
&\left.+ \frac{3\pi}{2}(x+4)^{1/2}\right) + 2x(\cos(x^2))^{-2}\sin(x^2).
\end{aligned}$$

If you find the linebreaks above incomprehensible, we can write the solution as

$$\frac{dy}{dx} = 3\tan^2 u \sec^2 u\left(\frac{1}{2\sqrt{\sin(2x)}}\cos(2x)\cdot 2 + \frac{3\pi}{2}(x+4)^{1/2}\right) + 2x(\cos(x^2))^{-2}\sin(x^2)$$

where $u = \sqrt{\sin(2x)} + \pi(x+4)^{3/2}$. It's also OK to treat the last term in the expression of y as $\sec(x^2)$, in which case you may automatically recognize that it has derivative $2x\sec(x^2)\tan(x^2)$, which is our last term above.