**MATH 1501G1/G2/G3** **Name:** _____
**Quiz #6** 4 October 2006

1. (5 points) Find the critical numbers and classify the extreme values of the function

$$f(x) = (x-1)^2(x-2)^2$$

with $x$ in the interval $[0, 4]$.

**Solution:** We first compute that

$$f'(x) = (x-1)^2 \cdot 2(x-2) + 2(x-1)(x-2)^2 = 2(x-1)(x-2)(x-1+x-2) = 2(x-1)(x-2)(2x-3).$$

The critical numbers of $f$ are the points in its domain where $f'$ is zero or undefined. The roots of $f'$ are 1, 2, and 3/2, all of which lie in the domain of $f$, so they are all critical numbers. We use the first derivative test to determine if $f$ has a local extremum at any of its critical numbers. When $x < 1$, $f'(x) < 0$. When $1 < x < 3/2$, $f'(x) > 0$. Also, when $3/2 < x < 2$, $f'(x) < 0$. Finally, when $x > 2$, $f'(x) > 0$. Thus, we have by the first derivative test that at $x = 1$ there is a local minimum of $f(1) = 0$, at $x = 3/2$ there is local maximum of $f(3/2) = 1/16$, and at $x = 2$ there is a local minimum of $f(2) = 0$. At the left endpoint of its domain, $f$ has value $f(0) = 4$ and since the derivative is negative near 0, we have that $f$ has an endpoint maximum of 4 at $x = 0$. At the right endpoint of its domain, $f$ has value $f(4) = 36$, and since the derivative is positive near $f$, we have that $f$ has an endpoint maximum of 36 at $x = 4$. Now looking at all of its extreme values, we see that $f$ has an absolute minimum of 0 at $x = 1$ and $x = 2$ and an absolute maximum of 36 at $x = 4$.