1. (5 points) Find the indefinite integral

$$\int \left[(3 - \csc x)^2 + \frac{e^x}{\sqrt{e^x + 1}} \right] dx$$

Solution: We begin by expanding the integrand and have that the indefinite integral we wish to find is

$$\begin{aligned} &\int (9 - 6\csc x + \csc^2 x + \frac{e^x}{\sqrt{e^x + 1}})\, dx \\ &\qquad = \int 9\, dx - \int \csc x\, dx + \int \csc^2 x\, dx + \int \frac{e^x}{\sqrt{e^x + 1}})\, dx \\ &\qquad\qquad = 9x - \ln|\csc x - \cot x| - \cot x + \int \frac{e^x}{\sqrt{e^x + 1}}\, dx. \end{aligned}$$

Now to deal with the last term, we use the substitution $u = e^x + 1$, which gives $du = e^x\, dx$. Thus, the final integral becomes

$$\int \frac{e^x}{\sqrt{e^x + 1}}\, dx = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C = 2\sqrt{e^x + 1} + C.$$

Therefore, the solution is

$$9x - \ln|\csc x - \cot x| - \cot x + 2\sqrt{e^x + 1} + C.$$