1. (5 points) Calculate the integral

$$\int x^2 \sin x \, dx.$$

Solution: We use integration by parts. We start with $u = x^2$ and $dv = \sin x \, dx$, in which case $du = 2x \, dx$ and $v = -\cos x$. Thus, we have

$$\int x^2 \sin x \, dx = -x^2 \cos x - \int (-\cos x) 2x \, dx = -x^2 \cos x + 2 \int x \cos x \, dx.$$

We again use integration by parts to deal with the remaining integral. This time, we have $u = x$ and $dv = \cos x \, dx$, which leads to $du = dx$ and $v = \sin x$. Therefore, we have

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx = x \sin x + \cos x + C.$$

Finally, the integral we were asked to calculate is

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2(x \sin x + \cos x + C) = -x^2 \cos x + 2x \sin x + 2 \cos x + C',$$

where $C' = 2C$.

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.