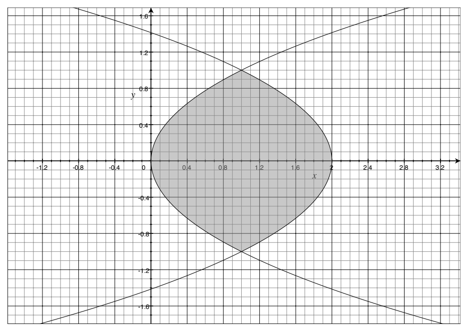
1. (5 points) Sketch the region Ω bounded by the curves and find the volume of the solid generated by revolving this region about the y-axis.

$$x = y^2, x = 2 - y^2$$

Solution: It is always helpful to draw a picture:



Since the question asks for solid generated by revolving about the y-axis we will integrate with respect to y. We also need to find the limits of integration which are the y values where the two equations intersect.

We have $x = y^2$, and $x = 2 - y^2 \Longrightarrow y^2 = 2 - y^2 \Longrightarrow 2y^2 = 2 \Longrightarrow y = \pm 1$

With that we can use the "Washer method about the y-axis" which is equation 6.2.6 in the Salas book.

In this case the biggest radius belongs to $x = 2 - y^2$ and the smallest radius belongs to $x = y^2$. So formula says:
$V = \pi \int_{-1}^{1} (2 - y^2)^2 - (y^2)^2\, dy = \pi \int_{-1}^{1} 2^2 - 2 * 2y^2 + (y^2)^2 - (y^2)^2\, dy = \pi \int_{-1}^{1} 4 - 4y^2\, dy =$
$4\pi \int_{-1}^{1} 1 - y^2\, dy \Longrightarrow 4\pi \left(y - \frac{y^3}{3}\right)$
Plug in the limits of integration

$4\pi \left(1 - \frac{1}{3} - (-1 - \frac{(-1)^3}{3})\right) = 4\pi \left(2 - \frac{2}{3}\right) = \frac{16\pi}{3} \Longrightarrow \boxed{V = \frac{16\pi}{3}}$

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.