

MATH 1113/2804 Final Exam

Spring 2007

Name: _____

GTid (9xxxxxxxx): _____

Instructor: Mitchel T. Keller

Course (1113/2804): _____

- There are 32 questions on this exam on 11 pages (not counting this coverpage). **Write your name at the top of each page.**
- Answer each question in the space provided; if you need additional space, you may write on the back of the page, but clearly indicate by the appropriate problem that you have work on the back.
- Be sure to explain your answers, as answers that are not accompanied by explanations/work may receive no credit.
- You are to complete this exam completely alone, without the aid of notes, texts, calculators, cellular telephones, personal digital assistants, or any other mechanical or digital calculating device.

By signing on the line below, you agree to abide by the Georgia Tech Honor Code, the principles of which are embodied by the Challenge Statement:

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Failure to sign this cover page will *not* be considered evidence of academic misconduct. However, **if the cover page is not signed, 5 points will be deducted from your raw total score on this exam.**

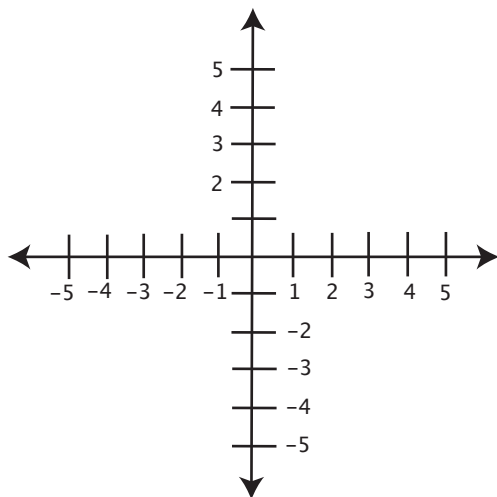
Student signature: _____

Questions 1 through 21 are for both MATH 1113 and MATH 2804. All subsequent problems are labelled to indicate the class.

There are 235 points possible on problems 1 through 21, which are required for both classes. There are 65 points possible on the MATH 1113 portion and 65 points possible on the MATH 2804 portion. The total number of points possible on this exam is therefore 300 points.

1. (10 points) Find the slope-intercept form of the line through the point $(2, 1)$ and perpendicular to the line with equation $y = 2x + 3$.

2. (10 points) The function $g(x) = |x + 2| - 1$ has parent function $f(x) = |x|$.
 - (a) Describe the sequence of transformations that should be performed on the graph of $f(x)$ to obtain the graph of $g(x)$.
 - (b) Sketch the graph of $g(x)$.



3. (10 points) Recall that the difference quotient for a function $f(x)$ is given by

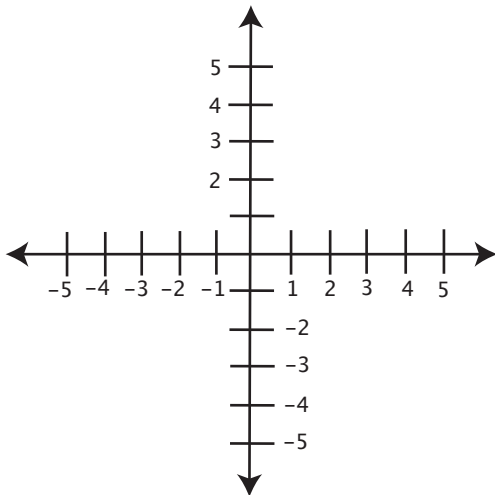
$$\frac{f(x+h) - f(x)}{h}$$

for $h \neq 0$. Let $f(x) = \frac{1}{x}$ and compute the difference quotient for this function, simplifying your answer as much as possible.

4. (10 points) Find the inverse function $f^{-1}(x)$ of the function $f(x) = \sqrt[3]{x+1}$.

5. (12 points) Consider the quadratic function $f(x) = x^2 - 6x + 11$.

- Write this function in standard form.
- Find the vertex.
- Sketch the graph of f .

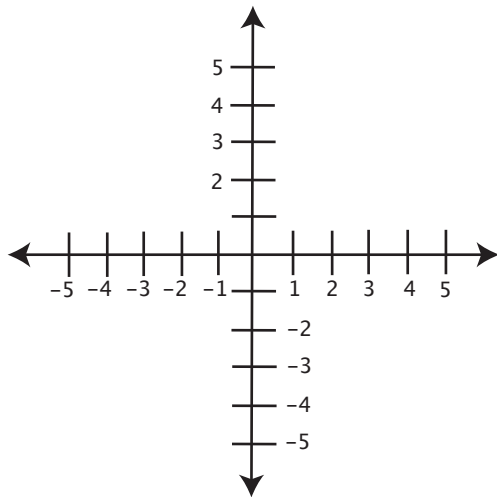


6. (10 points) Perform the division of polynomials $\frac{x^4 - 3x^2 + 2x - 1}{x + 2}$, writing your answer in the form $q(x) + \frac{r(x)}{d(x)}$.

7. (7 points) What information, if any, does the Leading Coefficient Test tell you about the shape of the graph of $s(x) = -x^{17} + 12x^{12} - 3x^3 + 2x - 1$? Use words or pictures in your answer, making sure to explain your answer. **Do not attempt a complete sketch of the graph of $s(x)$!**
8. (7 points) Write the complex number $\frac{3-i}{1+2i}$ in the form $a+bi$.
9. (14 points) Find all zeroes of the function $f(x) = 6x^3 + 13x^2 + x - 2$.
10. (14 points) Given that $2-i$ is a zero of $g(x) = x^3 - 3x^2 + x + 5$, find all zeroes of the function $g(x)$.

11. (16 points) Consider the function $f(x) = \frac{-4x + 1}{2x - 2}$.

- State the domain of f .
- Identify all intercepts of f .
- Find any vertical and horizontal asymptotes of f .
- Plot additional solution points as needed to sketch the graph of f .



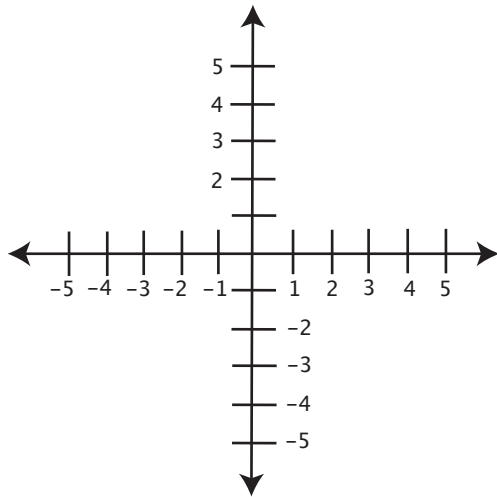
12. (10 points) Solve the inequality $2x^2 - x - 1 > 2$ and write the solution using interval notation.

13. (10 points) Expand $\log_3 \left(\frac{x^2 \sqrt[3]{z}}{y} \right)$. (You may assume all variables are positive.)

14. (10 points) Find the exact value of $\log_6 \sqrt{6^3} - 6^{2 \log_6 2}$.

15. (10 points) Solve for x in $e^{2x-1} = 3$.

16. (12 points) Describe the translations required to obtain the graph of $f(x) = 3^{-(x-1)} - 2$ from the graph of $g(x) = 3^x$ and then sketch the graph of $f(x)$.



17. (10 points) Solve the nonlinear system of equations below.

$$\begin{cases} -3x + y = -1 \\ (x+1)^2 - y = 2 \end{cases}$$

18. (10 points) Give the **form** of the partial fractions decomposition of the rational expression

$$\frac{x^2 + 12x - 1}{x^2(x-1)(x^2+3)}.$$

Do not solve for the constants!

19. (24 points) Perform the matrix operations indicated below, if it is possible to do so. If it is not possible, write “undefined” and briefly say why it is undefined.

(a) $3 \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -1 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 3 \end{bmatrix}$

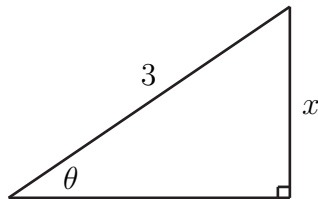
(c) $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 6 \\ -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

20. (9 points) Find the inverse of the matrix $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$. If the matrix is not invertible, say why.

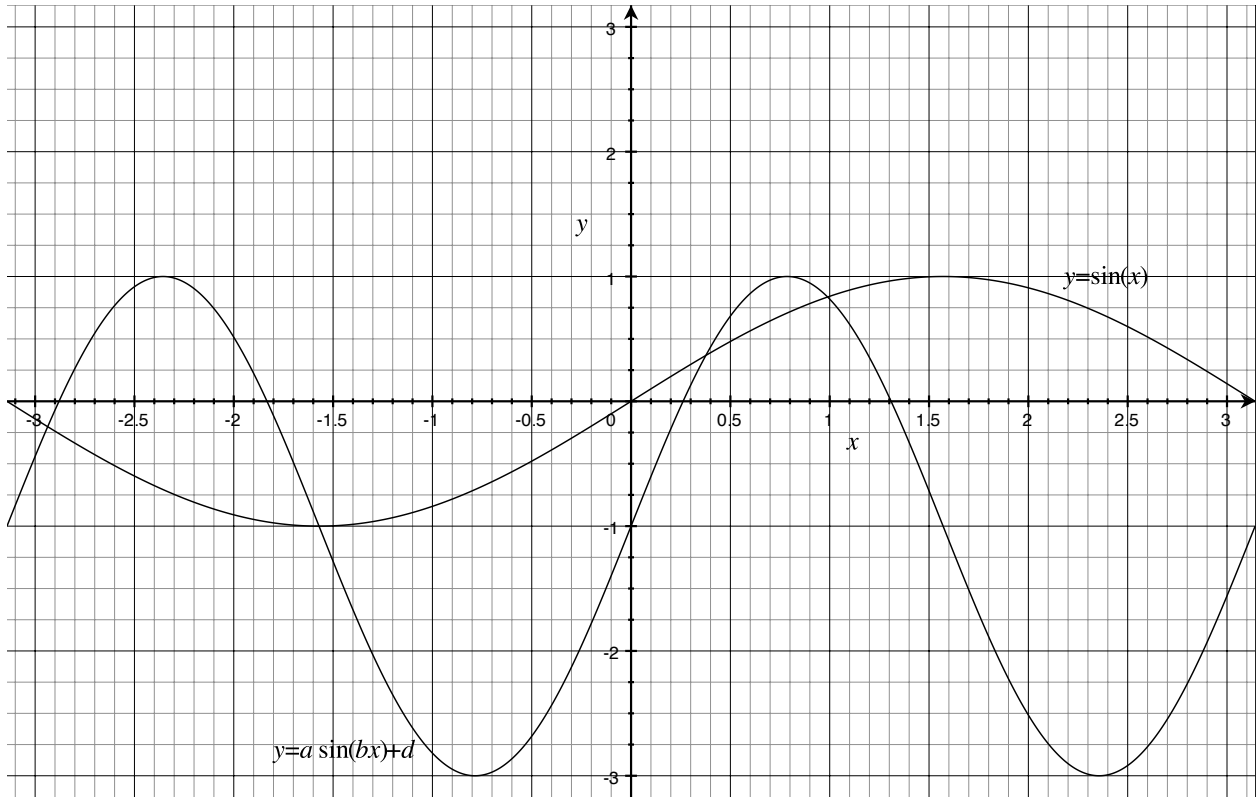
21. (10 points) Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$. Find $\det A$.

22. (5 points) **(1113)** Convert $\frac{7\pi}{9}$ radians to degrees.

23. (10 points) **(1113)** Solve for x in the triangle below given that $\theta = \frac{\pi}{12}$. [*Hint*: Use the fact that $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$.]



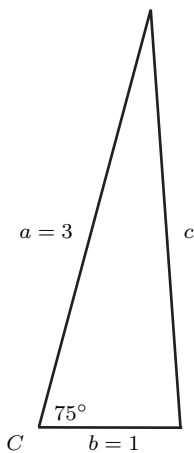
24. (10 points) **(1113)** The graph below shows one period of the graph of $y = \sin x$ (on the interval $[-\pi, \pi]$) as well as the graph of $y = a \sin(bx) + d$. Find the values of a , b , and d based on this graph.



25. (10 points) **(1113)** Given that $\sec t = -\frac{5}{4}$ and $\pi < t < \frac{3\pi}{2}$, find the values of the other five trigonometric functions.

26. (10 points) **(1113)** Find all solutions of the equation $\sin(2\theta) - \cos \theta = 0$ in the interval $[0, 2\pi)$. [*Hint*: Use an identity as your first step.]

27. (10 points) **(1113)** Consider a triangle in which angle C has measure 75° , $a = 3$, and $b = 1$. Find c , the length of the remaining side. [*Hint*: Use an identity to evaluate any trigonometric functions arising in your work at 75° .]



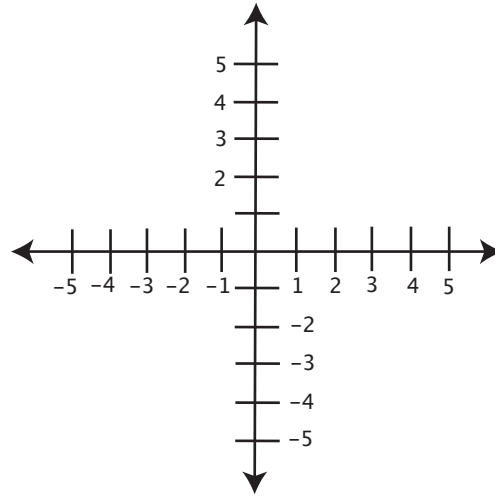
28. (10 points) **(1113)** Compute $(4\sqrt{2} - 4\sqrt{2}i)^{12}$ and write your answer in the form $a + bi$.

29. (5 points) **(2804)** Write the augmented matrix corresponding to the system of equations below. **Do not solve the system! Just write the matrix!**

$$\begin{cases} -x + 3y + 4z = 8 \\ 2x \quad \quad + 3z = \pi \\ x + 3y - z = 9 \end{cases}$$

30. (15 points) **(2804)** Sketch the solution of the system of inequalities below.

$$\begin{cases} 2x - y < 4 \\ 3x + y \geq 1 \end{cases}$$



31. (20 points) **(2804)** Solve the system of equations below.

$$\begin{cases} x + 2y + z = 10 \\ 2y - 2z = 8 \\ 2x + y + 5z = 8 \end{cases}$$

32. (25 points) **(2804)** Find the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} -1 & -2 & -2 \\ 3 & 7 & 9 \\ 1 & 4 & 7 \end{bmatrix}.$$