MATH 1113A Test I

Fall 2009

Name: ANSWERS .

GTid (9xxxxxxxx): _____

Group (Color and Rank): _____

Instructor: Mitchel T. Keller

- There are 8 questions on this exam on 4 pages (not counting this coverpage). **Write your name and group at the top of each page.**
- Answer each question in the space provided; if you need additional space, you may write on the back of the page, but clearly indicate by the appropriate problem that you have work on the back.
- Be sure to explain your answers, as answers that are not accompanied by explanations/work may receive no credit.
- You are to complete this exam completely alone, without the aid of notes, texts, calculators, cellular telephones, personal digital assistants, or any other mechanical or digital calculating device.

By signing on the line below, you agree to abide by the Georgia Tech Honor Code, the principles of which are embodied by the Challenge Statement:

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Student signature: _____

- You are strongly encouraged to **check your work** on questions on this test. You should **circle** the question number of any question for which you go back and check your work.
- In the space below, keep a record of the questions you work on **in the order you work on them**. For example, if you start with question 4, then work on question 2, then work on question 8, and then return to question 4, your list here would begin 4, 2, 8, 4, ...

1. (15 points) Let $f(x) = \dfrac{x^4+2x^3+x^2+2x+1}{x^2+1}$. Write $f(x)$ in the form $G(x) + \frac{R(x)}{x^2+1}$ where $\deg R < 2$.

$$\frac{x^4+2x^3+x^2+2x+1}{x^2+1} = \frac{x^2(x^2+1) - x^2 + 2x^3 + x^2 + 2x + 1}{x^2+1}$$

$$= x^2 + \frac{2x^3+2x+1}{x^2+1} = x^2 + \frac{2x(x^2+1) - 2x + 2x + 1}{x^2+1}$$

$$= x^2 + 2x + \frac{1}{x^2+1}$$

2. (7 points) Find the equation of the line passing through the point $(4,5)$ and perpendicular to the line $y-5 = 2(x-4)$.

The line has slope $-\frac{1}{2}$, since the given line's slope is 2. Thus, the equation is

$$y - 5 = -\frac{1}{2}(x-4)$$

3. (9 points) Let $T = \dfrac{10}{\sqrt{15+x}}$. Find functions f, g, and h, each simpler than the given function T, such that $T = f \circ g \circ h$. (By "simpler than", I mean none of f, g, and h may be the identity function $I(x) = x$.)

$$f(x) = \frac{10}{x}, \quad g(x) = \sqrt{x}, \quad h(x) = 15+x$$

4. Let $f(x) = -3x^2 + 9x + c$, where c is a real number.

(a) (10 points) The graph of $f(x)$ is a parabola. Find its vertex. (Your answer should contain c.)

Complete the square: $-3x^2+9x+c = -3(x^2-3x)+c$

$= -3\left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right) + c = -3\left((x-\frac{3}{2})^2 - \frac{9}{4}\right) + c$

$= -3\left(x-\frac{3}{2}\right)^2 + \frac{27}{4} + c$. The vertex is

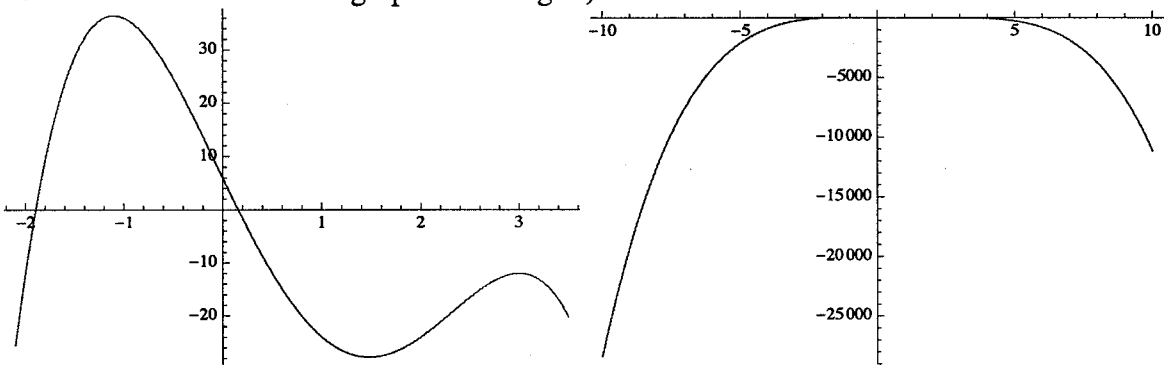
$\left(\frac{3}{2}, \frac{27}{4} + c\right)$

(b) (2 points) Is the vertex of this parabola a maximum, a minimum, or neither for $f(x)$? Why? Maximum since the coefficient on x^2 is negative.

(c) (4 points) Find the real number c so that the vertex of this parabola lies on the line $y = x$.

We need $\frac{3}{2} = \frac{27}{4} + c \Rightarrow c = \frac{3}{2} - \frac{27}{4} = \frac{6-27}{4} = \boxed{\frac{-21}{4}}$

5. Below are two pictures of the graph of a polynomial function $p(x)$. (The graph on the left is zoomed in further than the graph on the right.)



(a) (2 points) Use the graphs to estimate the real zeros of p.

The real zeros are approximately -1.9 and 0.18

(b) (3 points) Suppose that $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$. What do the graphs tell you about a_n? Why?

a_n is negative since the graph tends toward $-\infty$ for x near $+\infty$ and $-\infty$.

(c) (6 points) The graph tells you (at least) two things about $\deg p$. What are they and why?

The degree of p is at least 4 since the graph has three peaks and valleys. It must be even since the graph behaves the same way for x near $-\infty, +\infty$.

6. (8 points) Give the formula for a rational function that has $x = -2$, $x = 3$, and $x = 5$ as its **only** vertical asymptotes and $y = 0$ as its horizontal asymptote.

Need degree numerator less than degree denominator for $y=0$ as horizontal asymptote

$$f(x) = \frac{x^2}{(x+2)(x-3)(x-5)}$$

7. The graph of a function f with domain $[-1,3]$ is shown below (on the left). Suppose that $g(x) = 2f(-x) - 1$.

(a) (5 points) List (in order) the transformations performed on the graph of f go obtain the graph of g.

$f(-x)$: Reflect through vertical axis

$2f(-x)$: Vertical stretch by factor of 2

$2f(-x)-1$: Shift down one unit.

(b) (3 points) What is the domain of g?

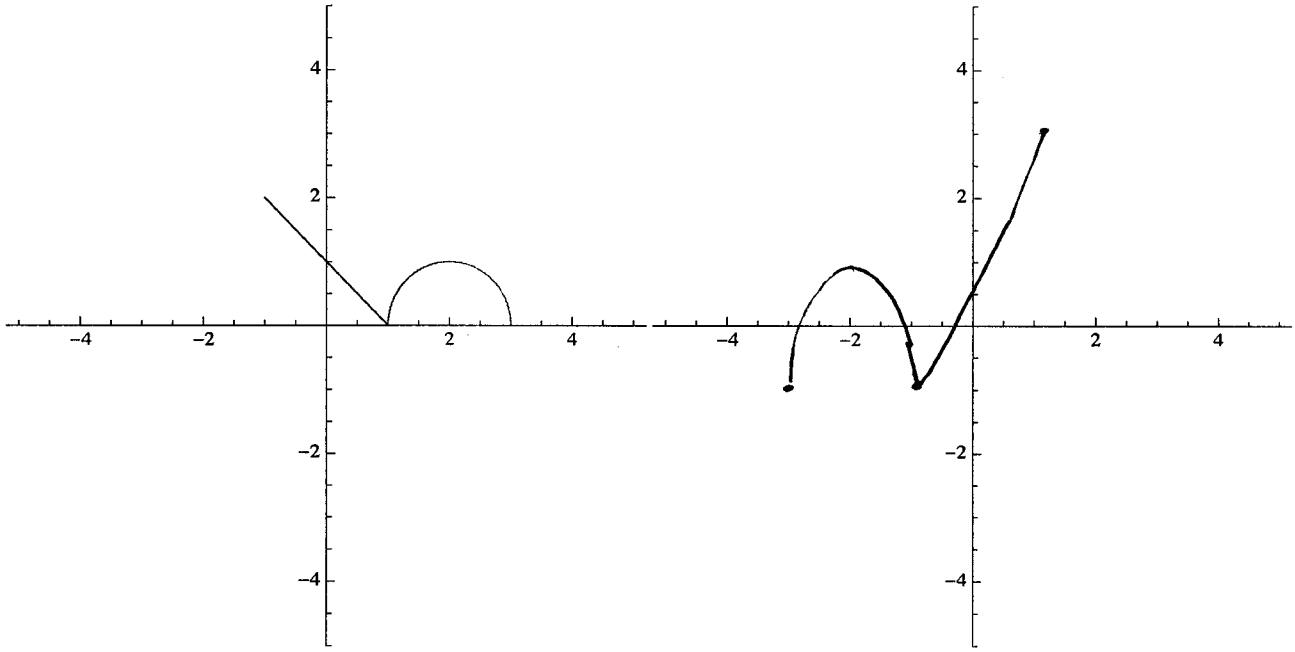
Only reflection through vertical axis involved, so $[-3, 1]$

(c) (3 points) What is the range of g?

Original range: $[0,2]$. ~~Scaling~~ Stretch makes it $[0,4]$.

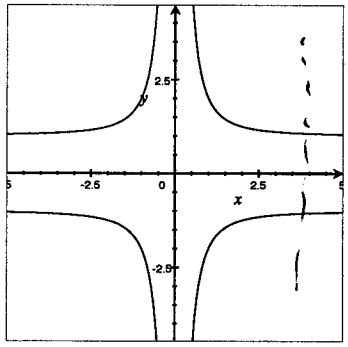
Shift makes it $[-1, 3]$.

(d) (5 points) Sketch the graph of g on the coordinate axes provided on the right below.



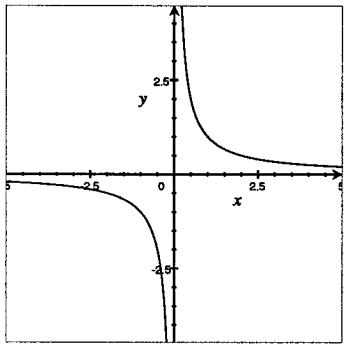
8. (8 points) For each graph below, state if it is the graph of an even function, the graph of an odd function, or neither. Be sure to explain your answers.

(a)



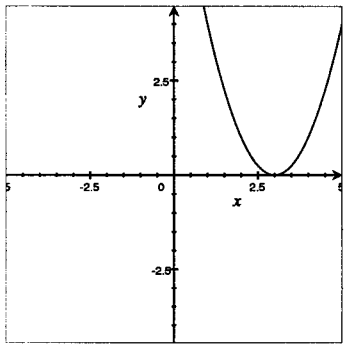
Neither: Fails vertical line test, so not even the graph of a function.

(b)



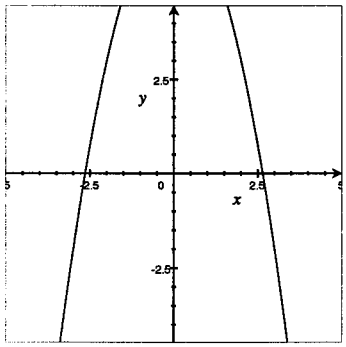
Odd: Symmetric about the origin.

(c)



Neither: Not symmetric about vertical axis or origin.

(d)



Even: Symmetric about vertical axis.