MATH 1113A Final Exam

Fall 2009

Name: _____

GTid (9xxxxxxxx): _____

Group (Color and Rank): _____

Instructor: Mitchel T. Keller

- There are 29 questions on this exam on 9 pages (not counting this coverpage). **Write your name and group at the top of each page.**
- Answer each question in the space provided; if you need additional space, you may write on the back of the page, but clearly indicate by the appropriate problem that you have work on the back.
- Be sure to explain your answers, as answers that are not accompanied by explanations/work may receive no credit.
- You are to complete this exam completely alone, without the aid of notes, texts, calculators, cellular telephones, personal digital assistants, or any other mechanical or digital calculating device.

By signing on the line below, you agree to abide by the Georgia Tech Honor Code, the principles of which are embodied by the Challenge Statement:

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Student signature: _____

- You are strongly encouraged to **check your work** on questions on this test. You should **circle** the question number of any question for which you go back and check your work.
- In the space below, keep a record of the questions you work on **in the order you work on them**. For example, if you start with question 4, then work on question 2, then work on question 8, and then return to question 4, your list here would begin 4, 2, 8, 4, ...

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Points:	5	6	12	9	5	8	6	5	12	10	10	12	6	6
Score:														

Part I—Trigonometry and Conic Sections

1. (5 points) Convert $\frac{5\pi}{12}$ radians to degrees.

2. For each equation below, say if it is the equation of a circle, an ellipse, a hyperbola, a parabola, or not the equation of a conic section. If it is not the equation of a conic section, say why it is not.

   (a) (2 points) $x^2 = \sqrt{y}$

   (b) (2 points) $\frac{(y+4)^2}{2} - \frac{(x-1)^2}{8} = 1$

   (c) (2 points) $(x+2)^2 + \frac{y^2}{4} = 1$

3. (12 points) If $\tan\theta = -\frac{1}{3}$ and $\cos\theta > 0$, find $\sin\theta$ and $\cos\theta$.

4. (9 points) Give a sine function (of the form $m\sin(kx-t)+d$) that has minimum value -4, maximum value 6, and period 3π.

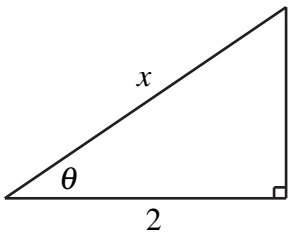
5. (5 points) Find the period of the function $-3\cos\left(\frac{x}{8}\right)$.

6. (8 points) Use an identity to find the exact value of $\sin 75°$.

7. (6 points) Evaluate $\tan\left(\frac{7\pi}{6}\right)$.

8. (5 points) Evaluate $\tan^{-1}(\tan 4\pi)$.

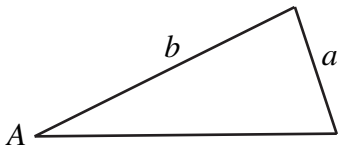
9. (12 points) Find the exact value of x in the right triangle below (not to scale!) if $\theta = \frac{\pi}{12}$. (Use an identity to find the exact value of any trigonometric functions.)



10. (10 points) Evaluate $\tan\left(\sin^{-1}\left(\frac{1}{4}\right)\right)$.

11. (10 points) Suppose that θ is an angle such that $\cos\theta = \frac{1}{5}$ and that $\sin\theta < 0$. Find the exact value of $\sin(2\theta)$.

12. (12 points) Suppose that $A = \frac{\pi}{6}$ and $a = 2$ in the triangle below (not to scale!). Find the value of b that makes the triangle a right triangle.



Part II—Comprehensive

13. (6 points) Suppose that $f(x) = \dfrac{\sqrt{x+2}}{(x+5)(x-4)}$ and that the domain of f is the set of real numbers for which this formula makes sense and produces a real number. Write the domain of f in interval notation.

14. (6 points) Write $\dfrac{2-i}{4+2i}$ in the form $a+bi$.

15. (8 points) Give a polynomial $p(x)$ (with real numbers as its coefficients) of degree 6 that has 2 and -1 as its **only** real zeros and $2+i$ as a complex zero or explain why such a polynomial cannot exist. (If you give a polynomial, you may write your answer in factored form.)

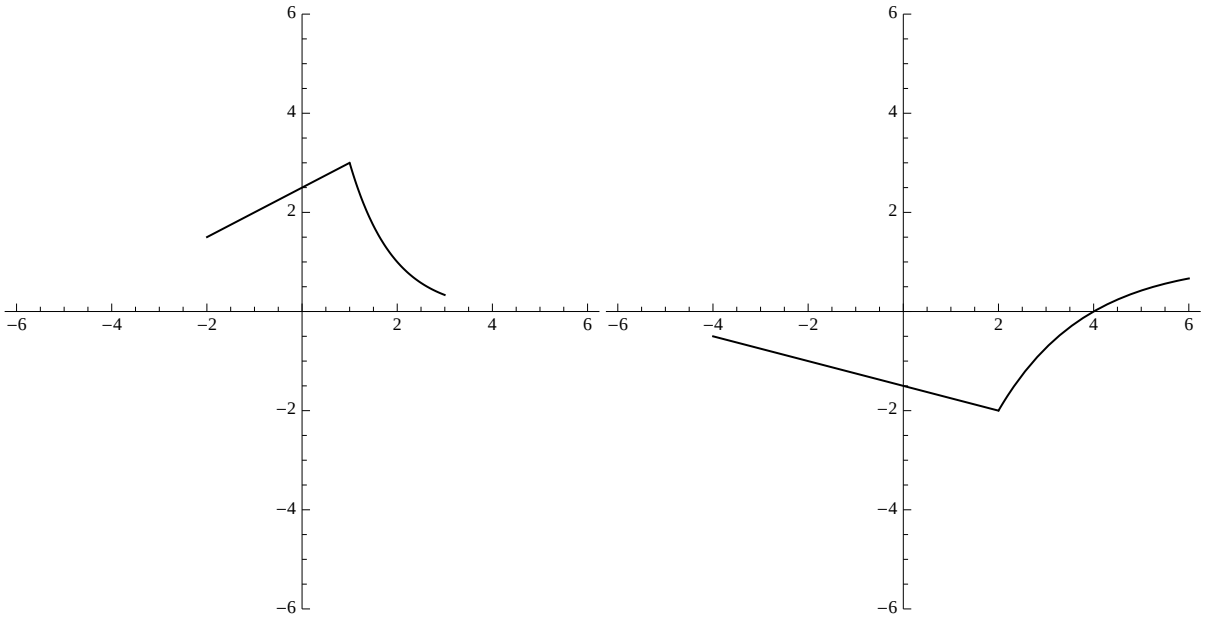
16. (8 points) Simplify the expression below completely. Your answer should not contain negative exponents or radicals.

$$\frac{\sqrt[4]{z^3}x^4y}{(x^3y^{-2})^3z^{1/2}}$$

17. The graph of a function f with domain $[-2,3]$ is shown below on the left. The graph of a function g that is obtained from f by transformations is shown below on the right.

    (a) (6 points) List (in order) the transformations performed on the graph of f go obtain the graph of g.

    (b) (6 points) Give a formula for $g(x)$ in terms of $f(x)$.



18. (6 points) Write $\frac{3}{4}\ln z + \ln(3x)$ as a single logarithm.

19. (10 points) Let $f(x) = (x-3)^2$. Let A be the point on the graph of f with x-coordinate 2, and let B be the point on the graph of f with x-coordinate $2+h$. Find the slope of the line passing through points A and B and simplify completely.

20. (6 points) Find t such that $2^{-3t+1} = \frac{1}{4}$.

21. (10 points) Find all values of x such that $\log_5(x+3) + \log_5(x-2) = 2$.

22. (8 points) Identify the horizontal and vertical asymptotes (if any) of the function f below.

$$f(x) = \frac{3x^3+2x-1}{(2x-1)(x+4)(x-7)} = \frac{3x^3+2x-1}{2x^3-5x^2-59x-28}$$

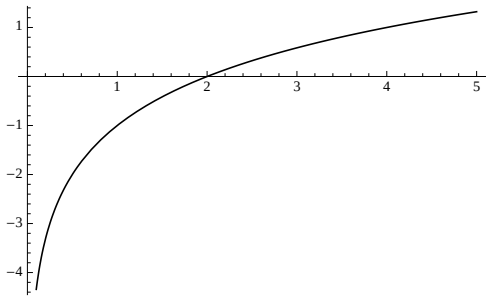
23. (17 points) Let $f(x) = \dfrac{x^4+7x^2+x+7}{x^2+5}$. Write $f(x)$ in the form $G(x) + \frac{R(x)}{x^2+5}$ where $\deg R < 2$.

24. (8 points) Let $T(x) = \sqrt{x^2+4}$. Find functions f and g, each simpler than the given function T, such that $T = f \circ g$. (By "simpler than", I mean neither f nor g may be the identity function $I(x) = x$.)

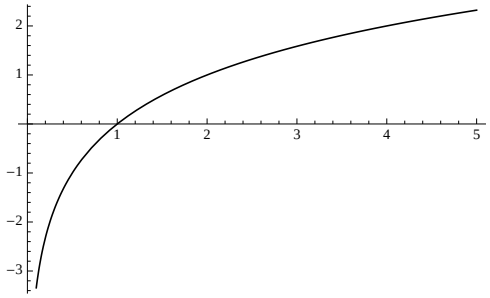
25. (8 points) Let $g(x) = \dfrac{5}{3x-1}$. Find a formula for the inverse function $g^{-1}(y)$.

26. For each graph below, say if it could or could not be the graph of $f(x) = \log_b x$ (not a transformation) and why. If you say a graph could be the graph of $\log_b x$, estimate b from the graph.

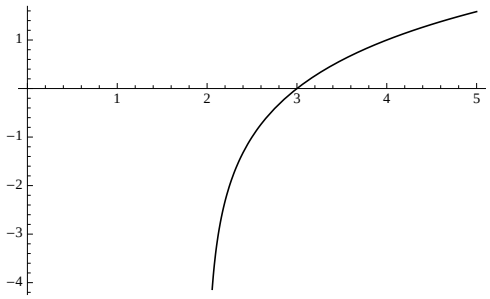
(a) (3 points)



(b) (3 points)

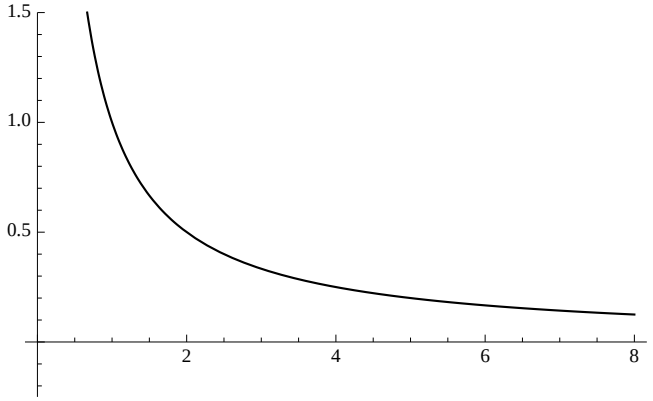


(c) (3 points)

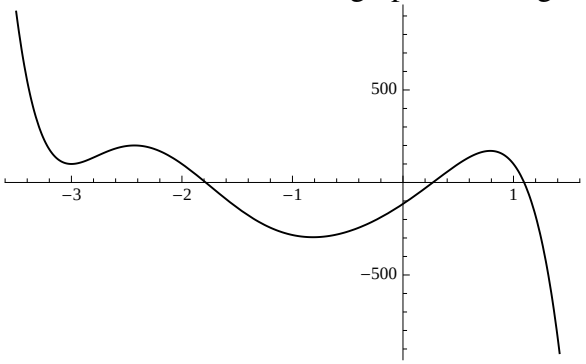


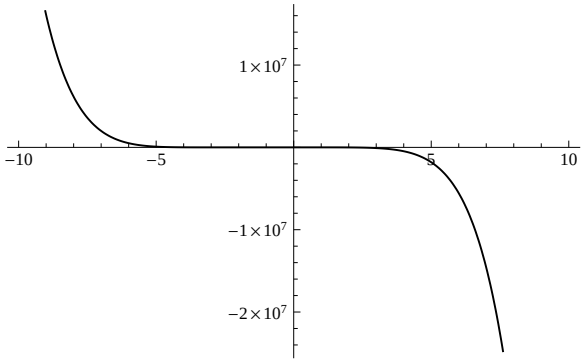
27. (10 points) Let $g(x) = 2x^2 + 8x + 1$. Find the minimum value of g. At what value of x does this minimum occur?

28. (6 points) What does $\ln 5 \approx 1.609$ tell you about a region under the curve $y = 1/x$? Sketch and describe that region as part of your answer.



29. Below are two pictures of the graph of a polynomial function $p(x)$. (The graph on the left is zoomed in further than the graph on the right.)





(a) (3 points) Use the graphs to estimate the real zeros of p.

(b) (6 points) The graph tells you (at least) two things about $\deg p$. What are they and why?

(c) (3 points) Suppose that $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$. What do the graphs tell you about a_n? Why?